TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 6 2016

Section 1 Calculator Free

Related Rates, Incremental Formula & Solving Differential Equations.

STUDENT'S NAME: ____

DATE: Thursday 1st September **TIME:** 25 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

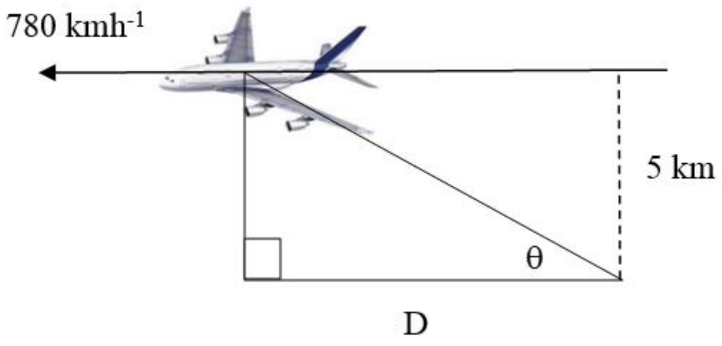
1. (7 marks)

(a) Determine an expression for $\frac{dy}{dt}$ given $y = e^{2x}$ and $\frac{dx}{dt} = 5$. [3]

(b) If $y = \sin(2x)$ and $\frac{dx}{dt} = 3$, evaluate $\frac{dy}{dt}$ when $x = \frac{\pi}{8}$. [4]

2. (10 marks)

You see a plane fly directly overhead at an altitude of 5 km. the plane is moving horizontally away from you at a constant speed of 780 $kmh^{-1}$with an angle of elevation of θ as shown.



(a) Show that the horizontal distance, D, between the plane and you is given by $D = \dfrac{5}{\tan(\theta)}$

[2]

(b) Determine the simplest expression for $\dfrac{dD}{d\theta}$. [3]

(c) Calculate the rate at which the angle of elevation is changing over time (in radians/hour) when $\theta = \dfrac{\pi}{6}$. [3]

(d) Is this a reliable measure of the rate, $\dfrac{d\theta}{dt}$, in the long run? [2]

3. (9 marks)

(a) Given the differential equation $\frac{1}{y^2}\frac{dy}{dx} = \frac{1}{x}$, solve for y given that when $x = e$, $y = 1$.

[4]

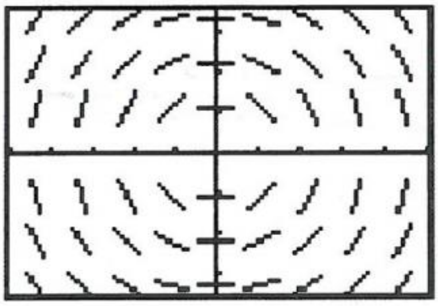
(b) Given that $\frac{dy}{dx} = \frac{1+y^2}{2xy}$, solve for y in terms of x, given that when $x = 1$, $y = -1$.

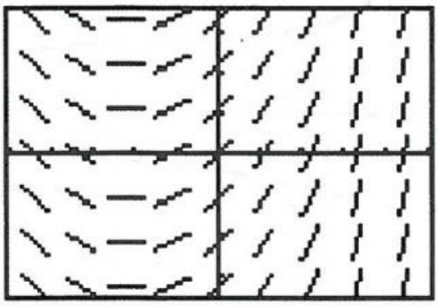
[5]

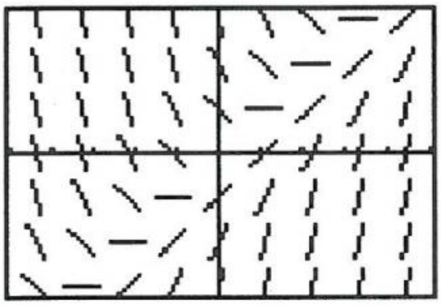
4. (4 marks)

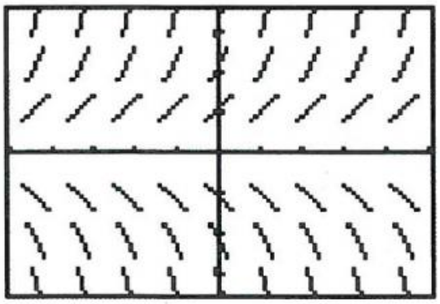
Match the slope field with the differential equation. Place the letter for the corresponding equation on the appropriate line

A. $\dfrac{dy}{dt} = \dfrac{1}{2}t + 1$ B. $\dfrac{dy}{dt} = t - y$ C. $\dfrac{dy}{dt} = y$ D. $\dfrac{dy}{dt} = -\dfrac{t}{y}$

______ 

______ 

______ 

______ 

End of Questions

TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 6 2016

Section 2 Calculator Assumed

Related Rates, Incremental Formula & Solving Differential Equations.

STUDENT'S NAME: ____

DATE: Thursday 1st September **TIME:** 25 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet retained from Section 1.

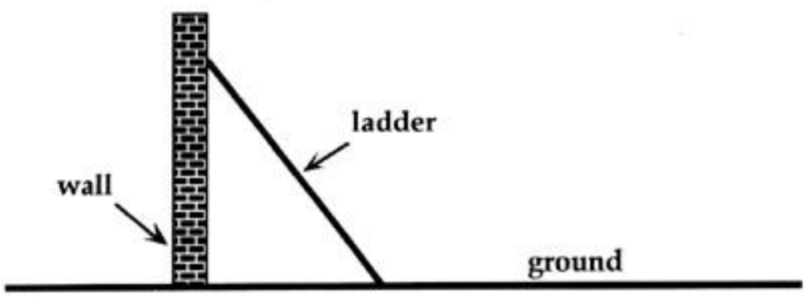
Special Items: Drawing instruments, templates, three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment).

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

5. (6 marks)

A 4 m long ladder, standing on horizontal ground, is leaning against a vertical wall. Its base is slipping away from the wall at a constant rate of 2 m/s. At what rate, correct to 2 decimal places, will the top of the ladder be slipping down the wall when the base is 1 m out from the wall?



6. (10 marks)

A tank contains 100 litres of brine with a concentration of 5 g/L. Fresh brine with a concentration of 20 g/L flows into the tank at a rate of 4 litres per minute. The concentration of the solution in the tank is kept uniform by constant stirring. The mixture flows out of the container at a rate of 4 litres per minute. The amount of salt at time t minutes is Q g.

Given the scenario is modelled by the differential equation: $\frac{dQ}{dt} = 80 - \frac{Q}{25}$

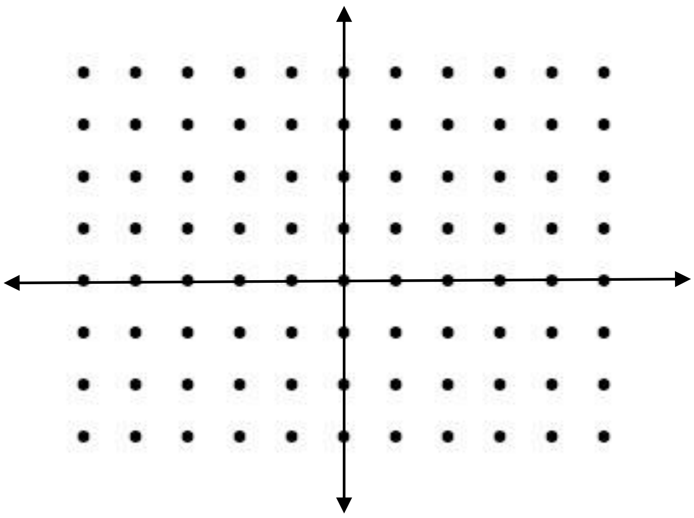
(a) Show that $Q = m - ne^{-kt}$, giving the values of the constants m, n and k. [6]

(b) Determine when the concentration in the mixture in the tank reaches 6 g/L. [2]

(c) Determine u and v, such that for any time t, $u \le Q < v$. [2]

7. (6 marks)

Sketch a slope field for $\frac{dy}{dx} = 2y$ [2]



(a) Use this slope field to sketch a solution through the point $(-1,1)$. [2]

(b) What is the particular solution to this differential equation with initial condition $y(-1)=1$? [2]

8. (3 marks)

The logistic differential equation, $\frac{dy}{dt} = ky\left(1 - \frac{y}{b}\right)$, has the logistic function, $y = \frac{b}{1 + Ae^{-kt}}$, as its solution.

(a) State the initial value, $y(0)$. [1]

(b) Identify the growth constant. [1]

(c) Determine the limiting value for y, otherwise known as the carrying capacity. [1]

9. (5 marks)

The radius of a circle increases from 20 cm to 20.1 cm.

(a) If A is the area of the circle, estimate the change in area by calculating $\frac{dA}{dr}$. [3]

(b) Calculate the actual area change, ΔA, and compare this with the result from part (a).

[2]

End of Questions